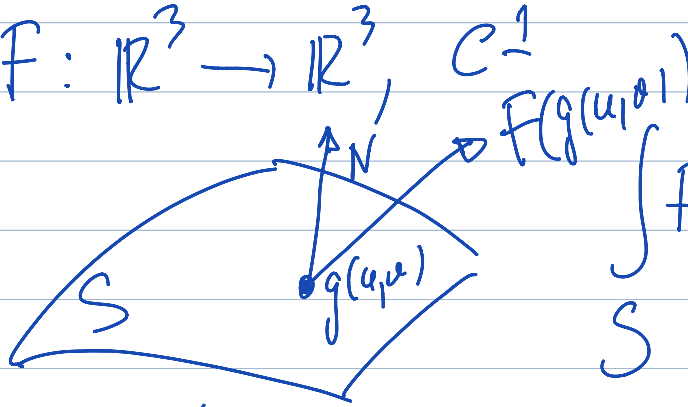
CDI-II – Prática 31/5/21

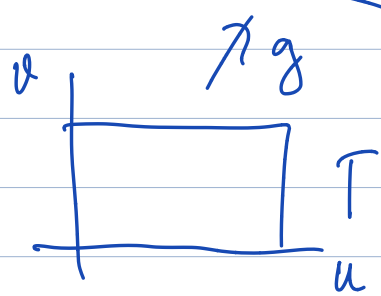
Ficha 12 – Ficha 13

F-12 – Fluxo definição e T. da divergência.

$F:\mathbb{R}^3 \to \mathbb{R}^3$, C^1



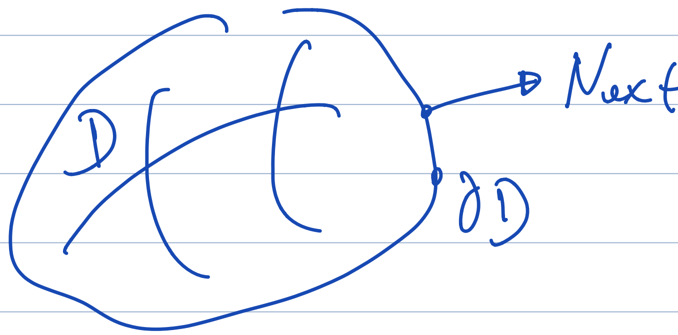
$$\int_S F\cdot N = \pm\iint_T F(g(u,v))\cdot \underbrace{D_u g \times D_v g}_{\text{sinal}}\, du\, dv$$



T. div:

$$\underset{\text{fluxo}}{\int_{\partial D} F\cdot N_{ext}} = \int_D \operatorname{div} F$$

$D \subset \mathbb{R}^3$, domínio regular



Ficha -12 :

4 — A: $z = x^2 + y^2 - 1$; $z < 0$, $y > 0$

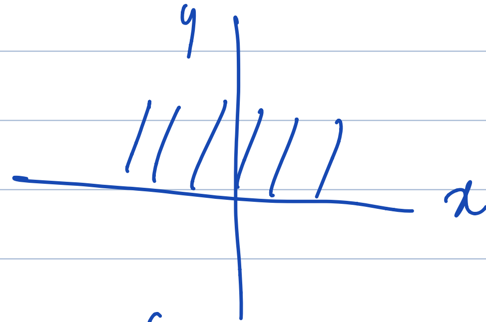
$n_z < 0$

$H(x,y,z) = (-y, x, z)$

$\int_A H \cdot n = ?$

1 — parametrizar A: (ρ, θ, z)

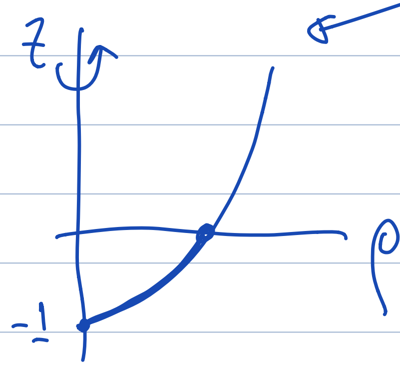
$\boxed{z = \rho^2 - 1} < 0$; $y > 0$



$y > 0 \Rightarrow 0 < \theta < \pi$

$$g(\rho,\theta) = (\rho\cos\theta,\ \rho \,\mathrm{sen}\,\theta,\ \rho^2 - 1)$$

$$T \begin{cases} 0 < \rho < 1 \\ 0 < \theta < \pi \end{cases}$$



$z = \rho^2 - 1$

$$\begin{array}{ccc} i & j & k \end{array}$$

2 - $\left.\begin{array}{l} D_\rho g = (\cos\theta,\ \mathrm{sen}\,\theta,\ 2\rho) \\ D_\theta g = (-\rho\,\mathrm{sen}\,\theta,\ \rho\cos\theta,\ 0) \end{array}\right\}$ tangentes

- $D_\rho g \times D_\theta g = (+2\rho^2\cos\theta,\ +2\rho^2\,\mathrm{sen}\,\theta,\ -\rho)$ normal

$n_z < 0$

3- $\int_A H \cdot n = -\int\int_T H(g(\rho,\theta)) \cdot \underbrace{D_\rho g \times D_\theta g}\, d\rho\, d\theta$

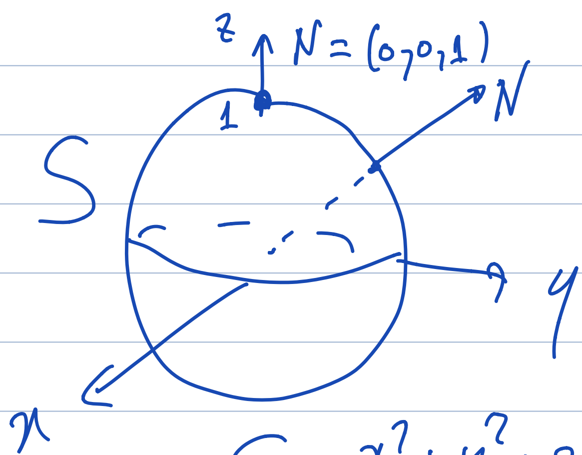
$= -\int_0^{\pi} \left(\int_0^1 (-\rho \operatorname{sen}\theta, \rho\cos\theta, \rho^2 - 1) \cdot (-2\rho^2\cos\theta, -2\rho^2 \operatorname{sen}\theta, +\rho)\, d\rho \right) d\theta$

$= -\int_0^{\pi} \left(\int_0^1 \rho(\rho^2 - 1)\, d\rho \right) d\theta$ etc. . .

—— " ——

6- $F(x,y,z) = h(r)(x,y,z)$ $\quad r = \|(x,y,z)\|$

$h:]0,+\infty[\to \mathbb{R}$, contínua.



$\int_S F \cdot N = ?$

$S: x^2 + y^2 + z^2 = 1 \to \boxed{r = 1}$

$$F(x,y,z) = h(r)\,(x,y,z) \leftarrow$$

em S : $F(x,y,z) = h(1)\,(x,y,z)$

$$N = \frac{(2x,2y,2z)}{\sqrt{4x^2+4y^2+4z^2}} = (x,y,z)$$

em S: $F \cdot N = h(1)\,(x,y,z)\cdot(x,y,z)$

$$F \cdot N = h(1).$$

$$\int_S F\cdot N = \int_S h(1) = h(1)\,\mathrm{vol}_2(S)$$

$$= 4\pi\, h(1).$$

——— u ———

8 - Volume de : $x^2+y^2 < z < 1$

$$\underbrace{\iiint_D \operatorname{div} F} = \iint_{\partial D} F\cdot N_{ext}$$

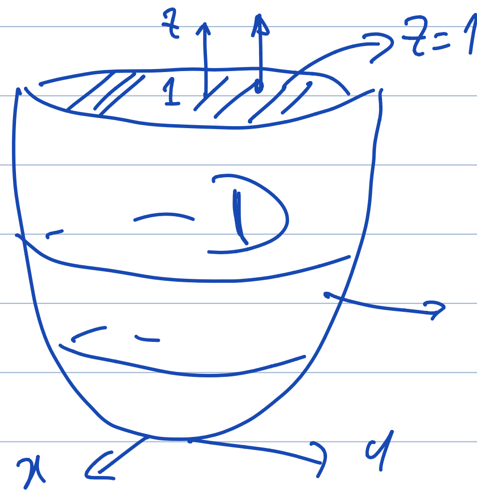
Fie $\operatorname{div} F = c \in \mathbb{R}$, atunci

$$c \, vol_3(D) = \iint_{\partial D} F \cdot N_{ext}$$

$$F(x,y,z) = (x,0,0) \longrightarrow \operatorname{div} F = 1 = c$$

$$F(x,y,z) = (0,y,0) \longrightarrow \operatorname{div} F = 1 = c$$

$$\boxed{F(x,y,z) = (0,0,z) \longrightarrow \operatorname{div} F = 1 = c}$$



$$\underbrace{x^2+y^2 < z < 1}_{D}$$

$$F(x,y,z) = (0,0,z)$$

$$vol_3(D) = \iint_{\partial D} F \cdot N_{ext}$$

$$\left.\begin{aligned} S_1 &: z = 1 \; ; \quad x^2+y^2<1 \\ N_{ext} &= (0,0,1) \end{aligned}\right\} F\cdot N_{ext} = 1$$

$$S: \quad z = x^2+y^2 \; ; \quad z<1$$

parametrizar, etc.

$$Vol_3(D) = \pi + \iint_S F\cdot N_{ext} \quad , \text{etc.}$$

---

7) $S: x^2+y^2 = 1+z^2 \quad ; \quad 0 \leq z < 1$

$n_z > 0$

$$F(x,y,z) = \left(2xyz,\ z^2 - zy^2,\ z(1-z)\right)$$

$$\int_S F\cdot n = ?$$

T. divergência: $\displaystyle\underbrace{\int_{\partial D} F\cdot N_{ext}}_{\text{Fluxo}} = \int_D \operatorname{div} F$

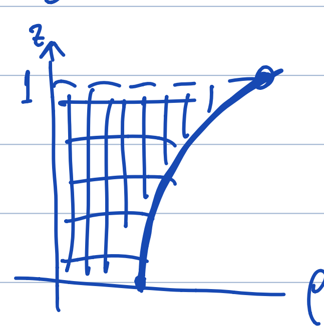
de S construir D tal que
$$S \subseteq \partial D.$$

$S \longrightarrow D$ aberto limitado regular

= 1 equação ... inequações

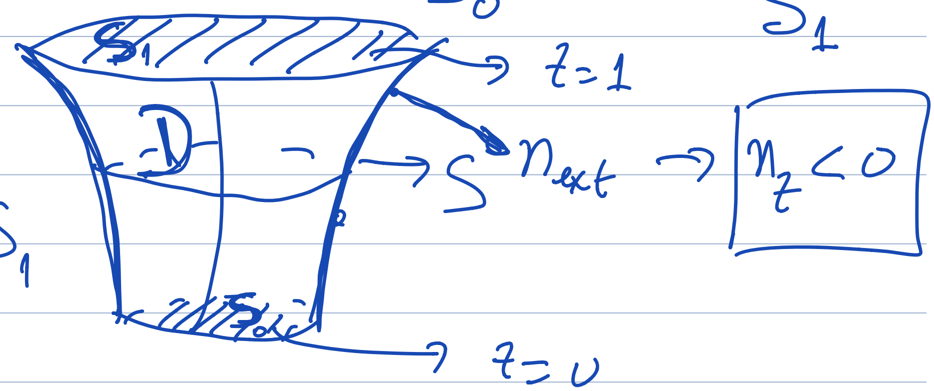
$$x^2+y^2=1+z^2 \quad \longrightarrow \quad x^2+y^2<1+z^2$$
$$0<z<1 \qquad\qquad\qquad 0<z<1$$

Nota: $\rho^2 < 1+z^2 < 1+1 = 2$ (limitado)



$$\partial D = S \cup \underbrace{\begin{Bmatrix} z=0 \\ x^2+y^2<1 \end{Bmatrix}}_{S_0} \cup \underbrace{\begin{Bmatrix} z=1 \\ x^2+y^2<2 \end{Bmatrix}}_{S_1}$$

$\partial D = S \cup S_0 \cup S_1$



$$\operatorname{div} F = \cancel{2yz} - \cancel{2yz} + 1 - 2z = 1 - 2z$$

$$S_0 : \left.\begin{aligned} N_{ext} &= (0,0,-1) \\ F(x,y,0) &= (\cdot\,,\cdot\,,0) \end{aligned}\right\} F \cdot N_{ext} = 0$$

$$\int_{S_0} F \cdot N_{ext} = 0 .$$

$$S_1 : \left.\begin{aligned} N_{ext} &= (0,0,1) \\ F(x,y,1) &= (\cdot\,,\cdot\,,0) \end{aligned}\right\} F \cdot N_{ext} = 0$$

$$\int_{S_1} F \cdot N_{ext} = 0 .$$

$$\iiint_D (1-2z)\, dx\, dy\, dz = \boxed{\iint_S F \cdot N_{ext}} + \underbrace{\iint_{S_0} F \cdot N_{ext}}_{=0} + \underbrace{\iint_{S_1} F \cdot N_{ext}}_{=0}$$

$$\iint_S F \cdot N_{ext} = \iiint_D (1-2z)\, dx\, dy\, dz$$

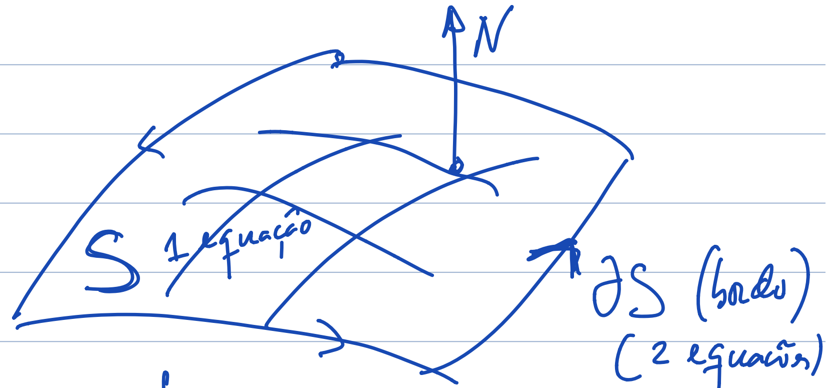
etc.

(ρ, θ, z)

Note: $N_{ext} = -N$

$$\iint_S F \cdot N = -\iint_S F \cdot N_{ext} = -\iiint_D (1-2z)\, dx\, dy\, dz$$

T. Stokes:



$F : \mathbb{R}^3 \to \mathbb{R}^3, \quad C^1$

$\boxed{S \text{ limitada}}$

$$\underbrace{\int_S \text{rot} F \cdot N}_{\text{Fluxo}} = \underbrace{\oint_{\partial S} F \cdot dg}_{\text{Trabalho}}$$

Ficha 13:

$$\text{rot} F \equiv \nabla \times F$$

1- F, S dados $\quad N_z < 0$

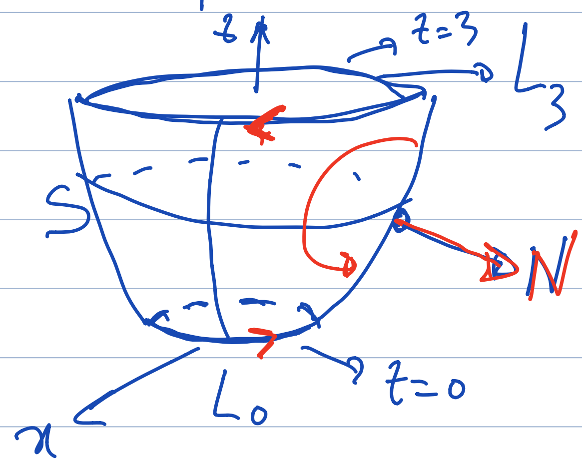
$$? \quad \int_S \nabla \times F \cdot N = \boxed{\int_{\partial S} F \cdot dg} \quad \text{a calcular!}$$

$$S \longrightarrow \partial S \ (\text{bordo})$$

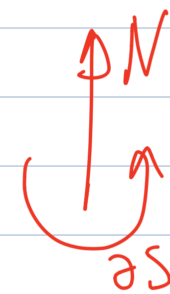
$$0 < z = x^2+y^2-1 < 3 \longrightarrow \left\{ \begin{array}{l} z=0 \\ z=x^2+y^2-1 \end{array} \right\} \cup$$

$\uparrow$ 1 eq.

$$\cup \left\{ \begin{array}{l} z=3 \\ x^2+y^2-1=3 \end{array} \right\}$$

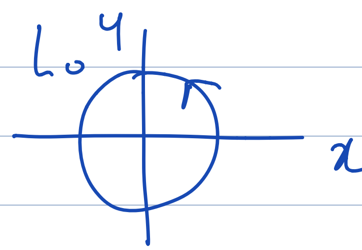


$$\partial S = L_0 \cup L_3$$



$$\int_S \mathrm{rot} F \cdot N = \int_{L_0} F \cdot dg + \int_{L_1} F \cdot dg$$

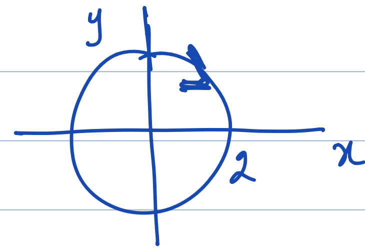
$$L_0 : \left\{ \begin{array}{l} z=0 \\ x^2+y^2=1 \end{array} \right.$$



$$g(t) = (\cos t, \operatorname{sen} t, 0), \quad 0 \le t \le 2\pi$$

etc

$$L_1 : \begin{cases} z = 3 \\ x^2 + y^2 = 4 \end{cases}$$



$$g(t) = (2\cos t, -2\,\text{sen}\, t, 3) \quad ; \quad 0 \le t \le 2\pi$$

etc.

$$\ldots \quad \int_a^b F(g(t)) \cdot g'(t)\, dt \quad \ldots$$